Control System Design for Power Converters
Today we will be talking about basics of control system design for power converters. The learning objectives are as follows. Importance of control system design, design of control systems and tricks and tips to improve the design.
Introduction to control systems.

Imagine you are driving a car and want to control the speed of the car. The speed of the car is the state variable.

The only thing that can be controlled is the pressure on the gas pedal. The pressure on the gas pedal is the manipulated input. The system acts in response to the manipulated input.

The desired speed is the reference for the system.

External factors like road gradient and wind drag are disturbances.

The feedback is the measured speed using speedometer.
One method to control the system is open loop implementation
The manipulated input is blindly set hoping the state variable will be at the desired value
It is easy to implement as no measurement devices of the actual state of the system is required
External factors like wind and gradient and variables like payload spoil the party
To prevent effects of external influences, leads to over designed passive system which adds bulk and cost.
Feedback is taken from using sensors which measure the desired output variable. This feedback is used to generate error between the reference. This error is then used to set the drive or the manipulated input. The block that takes in the error as input and generates the drive or the manipulated input is known as the controller. Closed loop systems can respond to changes in external factors. They reduce bulk and cost of passive components.

Closed loop control system design needs some mathematical analysis.
Closed loop control system

Plant \( G_p(s) \)

Feedback/sensor \( H(s) \)

Controller \( G_c(s) \)

\( V^* \)

\( c(s) \)

\( D(s) \)

\( V(s) \)
Closed loop control system design objectives

- Given: a system with input commands and output
- Control a physical quantity (state) to a varying desired value within a specified time
- Minimize effect of external influences
- Mitigate effect of measurement errors/noise
- Estimation as sensor replacement
Control system efficacy can be evaluated on 4 metrics:

- **Command tracking**: Ability of output to respond to varying input reference.
- **Disturbance rejection**: Ability to isolate output from variation in load.
- **Line Regulation**: Ability to isolate output from variation in input.
- **Noise rejection**: Ability to reject measurement noise/errors.
Closed loop control system response is frequency domain is shown here. Typically it looks like a low pass filter.

The cutoff frequency or bandwidth is the frequency over which the controller is not able to track changes in reference.
How to control a system

- A polynomial of s is selected for controller
- Typically s.Kd + Kp + Ki/s is used for controller
- Differentiation leads to noise amplification
  - **Caution**
- Estimators may be used for derivative terms

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How Do you Control the system and Why PID??

WHAT DO I HAVE IN HAND. Feed Back. I take the feed back either with amplification or attenuation and do a simple compare (P)

I LOOK AT THE Trend of my feedback with Past set of feedbacks and then control (Integrate)
I look at the rate of change of my feed back and control (Differentiate)

This Control Module is acting upon the system Transfer Function. Which means
Adding Gains to the system
Moving or adding Poles / Zeros for better response and stability
Buck converter

A buck converter is a power converter that converts higher voltage to a lower voltage using switches and inductor and capacitor. The basic algebraic equations are shown in the slide. The capacitor acts like an integrator for the current that flows into it. The output of the integrator is the capacitor voltage. The inductor acts like an integrator for the voltage that gets applied across it. The output of the integrator is the current. The voltage at the point where inductor and switches are connected is D.Vin

The goal is to control the output voltage to desired possibly changing values under varying conditions of line and load
Tricks and tips for insight

- Represent physical elements as integrations
- Natural characteristics of elements
  - E.g. To control Vo, Ic needs to be ‘manipulated’ by controller
  - Controller should emulate a current source
  - E.g. To control IL, VL needs to be manipulated by controller
  - Controller should emulate voltage source
- ‘information’ quantities should be interpreted as having physical units
- May lead to more than standard PID and more MATH
- Better performance

Designing control system gets simplified if characteristics of elements and respect for physical quantities
Observing the following rules should make the system design lot easier
Represent physical elements as integrations
Natural characteristics of elements
   E.g. To control $V_o$, $I_c$ needs to be ‘manipulated’ by controller
       Controller should emulate a current source
   E.g. To control $I_L$, $V_L$ needs to be manipulated by controller
       Controller should emulate voltage source
‘information’ quantities should be interpreted as having physical units
May lead to more than standard PID and more MATH
Better performance
The system subtracts Vo from the applied voltage
The difference is the inductor voltage which determines the current
The dynamics of current and voltage are cross coupled
To simplify system dynamics add Vo a priori and then apply new voltage
Decouples the current and voltage state variables
Controller output $V_x = G(V_{o^*} - V_o) + V_o$
Disturbance rejection/Load regulation

- Changing load causes output to change
- \( |\frac{I_o(s)}{V_o(s)}| \), amount of load to cause a unit change in output
- Performance METRIC
- The higher the value the STIFFER the system
- Different regions of the plot are determined by gains and system parameters
- Improves with passive component size \( \rightarrow \) COST
- Load decoupling WILL improve this metric

Changing load causes output to change
\( |\frac{I_o(s)}{V_o(s)}| \), amount of load to cause a unit change in output

Performance METRIC
The higher the value the STIFFER the system
Different regions of the plot are determined by gains and system parameters
Higher eigen value in general tends to lift the plot
Improves with passive component size however it leads to more COST
Load decoupling WILL improve this metric
The output of controller is Vx, which is the applied voltage to LC circuit. Needs to be converted to DSC modifyable parameter based on converter topology. E.g. for a buck converter D.Vin = Vx. This improves line regulation. DSC allows easy implementation of divide vs analog which can be implemented at a low frequency.
Closed loop system transfer function denominator polynomial is called as the characteristic equation.
Roots are known as eigen values / bandwidths.
Bandwidths are fixed by choice based on system specs.
Gains are determined by reverse calculation.
Voltage mode control (VMC)

- Only voltage feedback. No current sensors
- LC oscillatory roots cause poor dynamics
- D term is essential => poor noise rejection
- \( |\frac{I_o}{V_o}| = \frac{(s^2LC + sKd + (Kp + 1) + Ki/s)}{sL} \)
- \( \frac{V_o}{V_o^*} = \frac{G}{(s^2LC + sKd + (Kp + 1) + Ki/s)} \)
  where \( G = sKd + Kp + Ki/s \)

Only voltage feedback is taken
Roots are inherently oscillatory
PID controller is used for VMC to make roots non oscillatory
Typical command tracking and disturbance rejection plots
• f₁, > f₂, > f₃ hz are 3 bandwidths of characteristic equation \((s^2LC + sKd + Kp + Ki/s) = 0\), -2πf₁, -2πf₂, -2πf₃ are its roots

• Bandwidths are separated by a factor of 3 by choice

• f₃ is determined by settling time (Ki).

• f₂ is primary voltage loop BW (Kp)

• f₁ is differentiated voltage loop BW (Kd)

• Determine gains by solving 3 simultaneous eqns

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f₃ is determined by settling time (Ki).

f₂ is primary voltage loop BW (Kp)

f₁ is differentiated voltage loop BW (Kd)

determine gains by solving 3 simultaneous eqns
Current feedback is measured and an inner P current control is used
Current loop bandwidth is kept larger
Need extra current sensor
Better dynamics
Inner fast current loop allows fast current control
Easier to do control design
Current loop substitutes for D term
Gains are calculated from bandwidths
Peak current mode control

- Current loop implemented in hardware
- Outer PI voltage loop only sets the peak current reference
- Limits current to the reference
- Approx Ra → inf.

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\frac{V_o}{V_o^*} = \frac{(K_p + K_i/s)}{(sC + K_p + K_i/s)}
\]

\[
\frac{I_o(s)}{V_o(s)} = \frac{(sC + K_p + K_i/s)}{(sC + K_p + K_i/s)}
\]

Peak current mode control, the current controller is implemented in hardware. Current reference is generated by outer voltage loop. The actual current is cycle by cycle current limited using a comparator to the current reference value.
digital implementation

- Quantizers, Latches, Difference equations
- Bandwidth < 1/7th of sampling freq
- The term $\frac{Ki}{s}$ translates to $\frac{Ki.Ts.z^{-1}}{1-z^{-1}}$
- Ensure, $Ki.Ts.e$ is at least 1 for $e > E$
- Limitation of 16 bit DSC resolution. (esp. PFC)
- Normalization, Number format and saturation

Quantizers, Latches, Difference equations need to be taken care of in modeling
Highest Bandwidth should be less than 1/7th of sampling freq
The term $\frac{Ki}{s}$ translates to $\frac{Ki.Ts.z^{-1}}{1-z^{-1}}$
Ensure, $Ki.Ts.e$ may get truncated to 0 for some value of $e$. If the value of $e$ needs to lower $Ts$ may need to be increased
Limitation of 16 bit DSC resolution. (esp. PFC)
Normalization, Number format and saturation
Thank you!